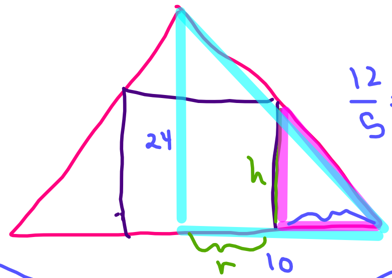
Maxmin3/ 12

8.3/ r=

5.9/ h=



$\frac{12}{5} = \frac{24}{10} = \frac{h}{10-r}$

$h = \frac{12}{5}(10-r)$

$V = \pi r^2 h$

$V = \frac{12}{5}\pi r^2 (10-r)$

$V = \frac{12}{5}\pi (10r^2 - r^3)$

$V' = \frac{12}{5}\pi (20r - 3r^2) = 0$

$r(20-3r) = 0$

$r = 0$ or $r = \frac{20}{3}$

$h = \frac{12}{5}\left(\frac{30}{3} - \frac{20}{3}\right)$

$\frac{\cancel{12}^{4}}{\cancel{5}_{1}}\left(\frac{\cancel{10}^{2}}{\cancel{3}_{1}}\right) = 8$

$h = 8''$

$r = \frac{20}{3}''$

is one of the last options you try

INTEGRATION BY PARTS!!!!! (6.3)

2 & out

We will need to do integrals like the following:

$$\boxed{\int \underset{dv}{x}\ \underset{u}{\ln x}\,dx,}\ \boxed{\int x^2e^x\,dx,}\ \text{and}\ \boxed{\int e^x \sin x\,dx.}$$

by part ~~t~~wice

In order to make that happen, we will need a new technique called integration by parts. We lovingly refer to it as VOO-DOO or is it VU-DU 🙂

We know:

$$\int \frac{d}{dx}[uv] = \int u\frac{dv}{dx} + v\frac{du}{dx} = uv' + vu'$$

So, it should make sense that if we integrate both sides we get.....

$$uv = \int u\cancel{v'}\tfrac{dv}{dx}\,dx + \int v\cancel{u'}\tfrac{du}{dx}\,dx = \int u\,dv + \int v\,du$$

If we move things around algebraically, we should be able to get

$$\boxed{\int udv = uv - \int vdu}$$

$$\int \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv} \qquad u = \ln x \qquad v = x$$

$$du = \frac{1}{x}dx \qquad dv = dx$$

$$\int \ln x\, dx = x \ln x - \int x \frac{1}{x} dx$$

$$x \ln x - x + C$$

INTEGRATION BY PARTS

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du$$

The hard part is deciding which part should be u and which part should be dv.

GUIDELINES FOR INTEGRATION BY PARTS

1. Try letting dv be the most complicated portion of the integrand that fits a basic integration formula. Then u will be the remaining factor(s) of the integrand.
2. Try letting u be the portion of the integrand whose derivative is a simpler function than u. Then dv will be the remaining factor(s) of the integrand.

dv has to be something you can integrate

Mostly...you to just practice and do a bunch! Oh Yeah!!!

EXAMPLE 1 Integration by parts

$\boxed{\int u\,dv} = uv - \int v\,du$

Evaluate $\boxed{\int xe^x\,dx}$.

normally we choose dv first, make sure it is something you CAN integrate

u= x v= e^x

du= $1\,dx$ dv= $e^x\,dx$

$$\int xe^x\,dx = xe^x - \int e^x\,dx$$

$$\int xe^x\,dx = \boxed{xe^x - e^x + C}$$

EXAMPLE 2 Integration by parts

Evaluate $\int x^2 \ln x \, dx$.

$$\int u \, dv = uv - \int v \, du$$

u= $\ln x$ v= $\frac{x^3}{3}$

du= $\frac{1}{x}dx$ dv= $x^2 dx$

$$\int x^2 \ln x \, dx = \frac{x^3}{3}\ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$$

$$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$$

EXAMPLE 3 An integrand with a single term

$\int u\,dv = uv - \int v\,du$

Evaluate

$$\int_0^1 \arcsin x \, dx$$

sin

6 + 0

-1

$u = 1-x^2$
$du = -2x\,dx$

u= arcsinx v= x

du= $\frac{1}{\sqrt{1-x^2}}dx$ dv= dx

$\int_0^1 \arcsin x\,dx = x\arcsin x + \frac{1}{2}\int_0^1 \frac{-2x}{\sqrt{1-x^2}}\,dx$

$x\arcsin x + \frac{1}{2}\int u^{-1/2}\,du$ $2u^{1/2}$

$= x\arcsin x + \sqrt{1-x^2}\Big|_0^1$

$\int_0^1 \arcsin x\,dx = 1\arcsin(1) + \sqrt{1-1} - (0+\sqrt{1}) =$
$\frac{\pi}{2} + 0 - 1 = \frac{\pi}{2} - 1$

$$\int_0^1 \arcsin x\,dx = \left[x \arcsin x + \sqrt{1-x^2}\right]_0^1 = \frac{\pi}{2} - 1$$

EXAMPLE 4 Repeated application of integration by parts

Evaluate $\int x^2 \sin x\, dx$.

$$\int u\,dv = uv - \int v\,du$$

u= x^2 v= $-\cos x$

du= $2x\,dx$ dv= $\sin x\,dx$

$$\int x^2 \sin x\,dx = -x^2\cos x - \int -2x\cos x\,dx$$

$$\int x^2 \sin x\,dx = -x^2\cos x + 2\int x\cos x\,dx$$

do by parts again!

$u=x$ $v=\sin x$

$du=dx$ $dv=\cos x\,dx$

$$\int x^2 \sin x\,dx = -x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right]$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

$$\int x^2 \sin x\,dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$$

EXAMPLE 5 Repeated application of integration by parts

Evaluate $\int e^x \cos 2x\, dx$. 2 and out

$$dv = e^x\,dx \Rightarrow v = \int e^x\,dx = e^x$$

$$u = \cos 2x \Rightarrow du = -2\sin 2x\,dx$$

so:

$$\int e^x \cos 2x\,dx = e^x \cos 2x + 2\int e^x \sin 2x\,dx$$

but...we have to do it again...

Making the same type of substitutions for the next application of integration by parts, we have

$$dv = e^x\,dx \Rightarrow v = \int e^x\,dx = e^x$$

$$u = \sin 2x \Rightarrow du = 2\cos 2x\,dx$$

which gives us:

$$\int e^x \sin 2x\,dx = e^x \sin 2x - 2\int e^x \cos 2x\,dx$$

Therefore, we have

$$\int e^x \cos 2x\,dx = e^x \cos 2x + 2e^x \sin 2x - 4\int e^x \cos 2x\,dx$$

$$+4\int e^x \cos 2x\,dx \qquad\qquad +4\int e^x \cos 2x\,dx$$

$$\frac{5}{5}\int e^x \cos 2x\,dx = \frac{1}{5}\left(e^x \cos 2x + 2e^x \sin 2x\right)$$

$$\int e^x \cos 2x\,dx = \frac{1}{5}\left(e^x \cos 2x + 2e^x \sin 2x\right) + C$$

Hwk #67
8.3/46,47
7.3/1-4,9,12,15,19,25,32